MA 405 Exam 2 10/25/19 (Total = 100 points) Show all work!!

NO CALCULATORS!!!

1. Answer each of the following True or False (5 pts. each):
   a) If $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k$ are vectors in a vector space V and
   $$\text{Span}(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k) = \text{Span}(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{k-1})$$
   then $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_k$ are linearly independent.
   b) The vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ form a basis for R^2.
   c) The vectors $(4\ 2\ 2)^T$,$(3\ 1\ 1)^T$, $(5\ 2\ 1)^T$ are linearly independent.
   d) None of the eight axioms in the definition of a vector space explicitly states the element $\mathbf{0}$ is unique. It is a consequence of the axioms.
   e) The dimension of the null space of a square matrix is positive if and only if the matrix is singular.
   f) The intersection of two subspaces of a vector space is also a subspace.
   g) If A is 5×7 and the dimension of its column space is 3, then the dimension of the null space of A is 2.
   h) If $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$ span $\mathbb{R}^n$, then they are linearly independent.
2. Choose the correct answer to each of the following (5 pts. each):
   A) The dimension of the null space of $\begin{pmatrix} 2 & 1 & 3 & 4 & 0 & 1 & 2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 6 & 8 & 0 & 2 & 4 \end{pmatrix}$ is a) 2; b) 3; c) 4; d) 5.
   B) If the vector $\mathbf{b}$ is in the column space of the matrix A, and if the columns of A are linearly dependent, then $A\mathbf{x} = \mathbf{b}$ a) has a unique solution; b) has more than one solution; c) may be inconsistent; d) satisfies none of the above.
3. (15 pts.) Let A be a 4×5 matrix. If $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_4$ are linearly independent and
   $$\mathbf{a}_3 = \mathbf{a}_1, \quad \mathbf{a}_5 = 2\mathbf{a}_1 - \mathbf{a}_2 + 3\mathbf{a}_4$$
   determine the reduced row echelon form of A.
4. Determine whether or not each of the following vectors in R^3 are linearly independent and compute the dimension of the subslpace of R^3 spanned by the vectors (10 pts. each).
   a) $\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$; b) $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$.
5. (15 pts.) Let $A = \begin{pmatrix} 1 & 2 & 0 & -1 & 2 \\ 0 & 0 & 0 & 4 & -3 \\ -2 & -4 & 0 & -1 & 2 \\ 1 & 2 & 0 & -2 & 1 \end{pmatrix}$. Find a basis for the row space of A, a basis for the column space of A, and a basis for $N(A)$.