MA 405 Exam I 9/27/19 (Total = 100 points) Show all work!

NO CALCULATORS!!!

1. Answer each of the following True or False (5 pts. each):
   a) An $m \times n$ homogeneous system of linear equations always has a nontrivial solution if $n > m$.
   b) For any $n \times n$ symmetric matrices A and B, AB is always symmetric.
   c) The matrix $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ is an elementary matrix.
   d) If B, $n \times n$ is nonsingular then AB is nonsingular for any $n \times n$ A.
   e) The matrix $\begin{pmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ is in reduced row echelon form.
   f) If B, $n \times n$, is nonsingular, then A, $n \times n$, is nonsingular if and only if A is row equivalent to B.
2. Choose the correct answer to each of the following (5 pts. each):
   A) Let I denote the $n \times n$ identity matrix. For any $n \times n$ B, $\begin{pmatrix} I & 0 \\ B & I \end{pmatrix}^{-1}$
   a) $= \begin{pmatrix} I & -B \\ 0 & I \end{pmatrix}$; b) $= \begin{pmatrix} I & 0 \\ B^{-1} & I \end{pmatrix}$; c) $= \begin{pmatrix} I & 0 \\ -B & I \end{pmatrix}$; d) does not necessarily exist.
   B) If A and B are both $n \times n$ and nonsingular then $((AB^{-1})^T)^{-1}=$
   a) $(BA^{-1})^T$; b) $B^T(A^{-1})^T$; c) $(A^{-1}B)^T$; d) $((B^{-1}A)^{-1})^T$.
3. (10 pts.) For the system

$$\begin{array}{rcrcrcr} 2x_1 & - & 2x_2 & + & 4x_3 & = & 8 \\ 2x_1 & + & 3x_2 & - & x_3 & = & 4 \\ 7x_1 & + & 3x_2 & + & 4x_3 & = & 7 \end{array}$$

   use Gaussian elimination on the corresponding augmented matrix to obtain an augmented in row echelon form. Is the system consistent? (Yes or No)
4. Use Gauss-Jordan reduction to find all solutions to (15 pts.):

$$\begin{array}{rcrcrcr} 3x_1 & & & + & 6x_3 & = & 9 \\ -2x_1 & + & x_2 & - & 3x_3 & = & -6 \end{array}.$$

5. Let $A = \begin{pmatrix} 1 & 3 & 1 \\ -2 & 2 & -1 \\ 2 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 5 \end{pmatrix}$, and $C = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$. Compute (5 pts. each):
   a) AB
   b) CB^TA
   c) $A - BB^T$
   d) $A^2 - 2A + I$.
6. (15 pts.) Find the inverse of A in problem 5 by first computing $\det(A)$ and $\operatorname{adj} A$.