MA 241 Final Exam August 8, 2003

(Total = 100 points) Show all work!!!!!

1. Find each of the following indefinite integrals *without* relying on the table (10 pts. each):

$$\text{a)} \int (2x+1)^{-1/2} dx \qquad \text{b)} \int \frac{\sin^{-1} x}{\sqrt{1-x^2}} dx$$

2. Evaluate each of the following integrals, each one being either definite or improper. Identify the improper integrals, and for each of these either find its value if it converges, or if it does not converge, say so explicitly (5 pts. each):

$$\text{a)} \int_2^5 \frac{1}{\sqrt{x-2}} dx \qquad \text{b)} \int_0^1 \frac{dx}{\sqrt{1+x^2}} \qquad \text{c)} \int_1^\infty \frac{1}{x^3} dx.$$

3. (10 pts.) The left, right, Trapezoidal, and Midpoint Rule approximations were used to estimate $\int_0^1 f(x)\, dx$ where, on the interval $[0, 3]$, f is positive, decreasing and concave up. The estimates were 3.1546, 3.1348, 3.1743, and 3.1592, and the same number of subdivisions were used in each case. Which rule produced which estimate?

4. (5 pts.) A force of 10 lb is required to hold a spring stretched 1/3 ft beyond its natural length. How much work is done in stretching it from its natural length to 1/2 ft beyond its natural length?

5. Find the solution of each of the following differential equations that satisfy the given initial conditions (5 pts. each):

$$\text{a)} \frac{dy}{dx} = x^2 y, \quad y(0) = 2 \qquad \text{b)} \frac{dy}{dx} = y^2 + 1, \quad y(1) = 0.$$

6. Find the solution to each of the following initial-value problems:

a) (5 pts.) $y'' - 4y' + 3y = 0$, $y(0) = 2$, $y'(0) = 1$

b) (10 pts.) $y'' + 6y' + 13y = \sin x$, $y(0) = 0$, $y'(0) = 1$.

7. Determine whether each of the following series is convergent or divergent. If it is convergent, find its sum (5 pts. each):

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{3^n}$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}.$

8. Find the radius of convergence and interval of convergence of each of the following power series (5 pts. each):

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{n}$ b) $\displaystyle\sum_{n=0}^{\infty} \frac{n(x-3)^n}{2^{n+1}}.$

9. (5 pts.) Evaluate $\displaystyle\int \frac{e^x}{x} dx$ as an infinite series.