MA 241 - Exam 4 July 31, 2003
(Total = 100 points)
Show all work!!!!

1. (15 pts.) Find the solution to the following initial-value problem using the method of undetermined coefficients

$$y'' + y = \sin x, \quad y(0) = 0, \quad y'(0) = 1.$$

2. (15 pts.) Using the method of variation of parameters, find the general solution to

$$y'' + 2y' + y = \frac{e^{-x}}{1 + x^2}.$$

3. (20 pts.) A spring with a mass of 2 kg has a damping constant 14, and a force of 6 N is required to keep the spring stretched .5 m beyond its natural length. The spring is stretched 1 m beyond its natural length and then released with zero velocity. Find the position of the mass at any time t.

4. Determine whether each of the following sequences converges or diverges. If it converges, find the limit (10 pts. each):

a) $\{(-1)^n \sin(1/n)\}$ b) $\{\ln(n+1) - \ln n\}$.

5. Determine whether each of the following series is convergent or divergent. If it is convergent, find its sum (10 pts. each):

a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{e^{2n}}$ b) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2}$.

6. (10 pts.) Find the values of p for which the following series is convergent:

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}.$$