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A tribute to P.R. Krishnaiah

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ABSTRACT

The authors reminisce on their association with P.R. Krishnaiah, renowned professor of statistics at the University of Pittsburgh and founding editor of the *Journal of Multivariate Analysis*. They recount their individual associations with him, mainly involving the behavior of eigenvalues of random matrices, and outline two areas of applied work he performed with one of the authors.

1. Introduction

Paruchuri R. (P.R.) Krishnaiah was considered a great leader in the field of statistics, especially multivariate analysis. He devoted his academic career in the pursuit of knowledge and was a major influence in this area of statistics. He founded the famous **Center for Multivariate Analysis** at the Department of Mathematics and Statistics, University of Pittsburgh in 1982 and in 1971 created the prestigious *Journal of Multivariate Analysis* as the founding editor in chief. Based on his wide knowledge in statistics and familiarity of the literature, his activities were spread throughout the world. He has contributed a great deal, including the area involving limit theorems on the eigenvalues of random matrices as the dimensions increase. The authors of this paper became directly associated with Professor Krishnaiah's work on large dimensional random matrices. They pay tribute to this wonderful man by first recalling their fond remembrance of him, and then outlining significant contributions he gave to image reconstruction and to general information criteria (GIC) in model selection (note: "ZB" will refer to the first author, "JS" to the second author).

2. Association with Professor Krishnaiah

The recollection spans the years 1978–1987, beginning in the winter of 1978 when JS visited the Department of Mathematics and Statistics at the University of Pittsburgh for a job interview. He gave a talk on his work on the eigenvalues and eigenvectors of random matrices, the former stemming from his Ph.D. thesis and appearing in print [11]. Professor Krishnaiah was interested in his work and gave him a copy of a recent manuscript by Dag Jonsson to review for possible publication in JMVA [13]. From this manuscript JS became aware of a paper [15] published in 1967. At this point in time there were four papers, including a recent publication by Kenneth Wachter [26], dealing in the limiting

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behavior of eigenvalues of a random matrix, suitable for applications in multivariate analysis, as the dimension of the matrix increases. The assumptions on the entries of the matrix vary, but there is one matrix ensemble common to all of them, the sample covariance matrix arising from the identity matrix being the population matrix. In its basic form the matrix is of the form $B_p = (1/n)X_pX_p^T$, where X_p is $p \times n$ containing i.i.d. standardized entries. Thus B_p can be viewed as the sample matrix resulting from n samples (the columns of X_p) of a random vector having the identity matrix as its population matrix. The main result is on F_p, the empirical distribution function (e.d.f.) of the eigenvalues of B_p ($F_p(x) = (1/p)\{$no. of eigenvalues of $B_p \le x\}$). It states: if $n = n(p)$ and $p/n \to y > 0$, as $p \to \infty$, then, with probability 1, F_p converges weakly to what is now known as the Marčenko–Pastur distribution F_y, where for $y \le 1$,

$$F_y'(x) = f_y(x) = \frac{\sqrt{(x-a(y))(b(y)-x)}}{2\pi yx}, \qquad x \in (a(y), b(y)), \qquad a(y) = (1-\sqrt{y})^2, \quad b(y) = (1+\sqrt{y})^2, \ 0 \text{ otherwise},$$

and, for $y > 1$, F_y has the above density on $(a(y), b(y))$ and mass $1 - 1/y$ at 0 (note: a.s. convergence is only shown in [26], the others show convergence in probability).

It is believed that these four papers [11,13,15,26] and this common result is the main source of inspiration for Professor Krishnaiah to pursue extending multivariate statistics to large dimensional random matrices, essentially exploiting whatever limiting results are obtained on the eigenvalues of random matrices as the dimension approaches infinity to situations where the dimension of the matrix is high. For a sample covariance matrix these limiting results will be useful when the number of samples needed to provide conventional multivariate tools would be unattainable.

During JS's postdoctoral in the Division of Applied Mathematics at Brown University, Professor Ulf Grenander, his Ph.D. advisor, became curious as to the limiting property of the largest eigenvalue of B_P. After performing some simulations on increasing p, Professor Grenander became convinced that the largest eigenvalue converges a.s. to $b(y)$ as $p \to \infty$, and presented to Stuart Geman and JS a plan how to prove it. Professor Geman worked on his suggestion and came up with a proof [10]. At the same time JS searched through the literature to find any information on the distribution of the largest eigenvalue of B_p. He discovered the paper written by Professor Krishnaiah and T.C. Chang [14] which gives the distribution of the extreme eigenvalues of a Wishart matrix, that is, when the entries of X_p are i.i.d. standardized Gaussian. JS attempted to obtain from the formula for the distribution of the largest eigenvalue information as p increases, but failed to do so. But from that paper JS began to learn of Professor Krishnaiah's research abilities and standing in the mathematical and statistical communities.

The next time JS met up with Professor Krishnaiah was in Philadelphia, PA at an Institute of Mathematical Statistics meeting, in May 1981. He had invited JS to give a talk on the eigenvalues and eigenvectors of large dimensional random matrices. JS had been working on comparing the distribution of the orthogonal matrix of eigenvectors of the sample covariance matrix B_p to the case where B_p is Wishart distributed. In this case it is well-known that the matrix of eigenvectors is Haar distributed in the orthogonal group, $\mathcal{O}_p$ of $p \times p$ orthogonal matrices. The question raised by JS is how much is the distribution of the matrix of eigenvectors on $\mathcal{O}_p$ related to that of Haar measure when the entries of X_p is not $N(0, 1)$? This work resulted in several papers [18,19,21,23,24], two appearing in JMVA.

At the meeting Professor Krishnaiah asked JS to visit his department in Pittsburgh in August 1981 to lecture on his results on eigenvector behavior. During this visit Professor Krishnaiah told JS of his plan to work on eigenvalue behavior of large dimensional random matrices, along with Yong-Quan Yin (YY), a recent graduate student of the University of Pittsburgh from China. He was one of the first to be allowed to leave from China after the cultural revolution subsided. Before the cultural revolution he was a statistics professor, but had not gotten his Ph.D. The plan was to get his degree under Professor Krishnaiah. YY and JS became fast and life-long friends. YY and JS saw each other on several occasions, mainly since YY's daughter was a graduate student in the Statistics Department at the University of North Carolina at Chapel Hill, and later worked in the Research Triangle Park, all close to where JS lived.

From this point on, a professional and personal relationship developed between Professor Krishnaiah and JS The work between Professor Krishnaiah and YY on random matrices began to flourish. All their work was sent to JS and he was asked to referee several of them for possible publication in JMVA. Their first paper was published in JMVA in 1983 [29]. It dealt with the limiting e.d.f. of the eigenvalues of the product $(1/n)B_pT_p$ of two independent random matrices. Here B_p is as above and Wishart (entries of X_p being $N(0, 1)$ random variables), T_p is assumed to be $p \times p$ symmetric, and, with G_p denoting the e.d.f. of its eigenvalues, the moments $\int x^k dG_p(x)$ converge in probability to H_k which satisfy the Carleman sufficiency condition $\sum_{k=1}^{\infty} H_{2k}^{-1/2k} = \infty$. The conclusion is that, if $p/n \to y$ finite as $p \to \infty$, the e.d.f. of the eigenvalues of $(1/n)B_pT_p$ converges in probability weakly to a nonrandom distribution function. The proof relies on the method of moments, involving intricate combinatorial arguments using graph theory.

This paper allowed for the study of two types of random matrices found in multivariate analysis. Indeed, when T_p is nonnegative definite and nonrandom then one can consider the ensemble $(1/n)T_p^{1/2}X_pX_p^TT_p^{1/2}$, where $T_p^{1/2}$ is any symmetric square root of T_p. This matrix can be viewed as the sample covariance matrix of n samples of a Gaussian distributed random vector with mean 0 and population matrix T_p. It can also be viewed as a multivariate F matrix when $T_p = ((1/n')X_p'X_p'^T)^{-1}$ with X_p' $p \times n'$, $p < n'$ consisting of i.i.d. standardized entries, as long as it is known that the smallest eigenvalue of $B_p' = (1/n')X_p'X_p'^T$ remains positive in some probabilistic sense. This latter ensemble is handled in Professor Krishnaiah and YY's followup paper [27], which also appeared in JMVA and had a third author, ZB who was a student of YY in China and at the time was a graduate student at the China University of Science and Technology.

During most of the 1980s JS devoted most of his time on developing results on the eigenvectors of B_p, but was side-tracked on a few occasions to deal on eigenvalue issues. Two of these issues stem from [27]. The first involved the following result: for B'_p Wishart distributed, with $p/n' \to y \in (0, 1)$, let $\underline{\lambda}_p$ denote the smallest eigenvalue of B'_p. Then for any $y \in (0, 1/2)$ there exist positive ε_0, C, D, α such that for all $\varepsilon \in (0, \varepsilon_0]$ and all p

$$P(\underline{\lambda}_p \le \varepsilon) \le CD^p \varepsilon^{\alpha p}.$$

From this and the Borel–Cantelli lemma one can conclude $\liminf_{p\to\infty} \underline{\lambda}_p \ge \varepsilon$ a.s. for any $\varepsilon \in (0, \varepsilon_0]$ for which $D\varepsilon^{\alpha} < 1$. This allowed for the result on the eigenvalues of multivariate F matrices using the result in [29], at least in the case where the limiting p/n' is less than 1/2. JS discovered after reading the manuscript of the paper that the above bound holds for all $y \in (0, 1)$ after a slight modification in its proof. He sent off a letter to YY describing the modifications. After seeing that the final version of the paper did not contain his modifications, he wrote them up, submitted to JMVA and the paper was published a year later [20].

The second issue is that no form of the limiting empirical distribution of the eigenvalues of the multivariate F matrix is given in [27]. However the moments of the limiting e.d.f. of the eigenvalues of $(1/n)B_pT_p$ are explicitly given in [29] in terms of the limiting moments H_k. From this JS was able to find explicitly the limiting e.d.f. of the eigenvalues of the multivariate F matrix [22].

Professor Krishnaiah and YY also tackled another ensemble of random matrices fundamental to multivariate statistics [5,30,31]. It is of the form $A_p = (1/n)X_pX_p^T T_p$, with X_p, T_p independent, T_p $p \times p$ symmetric nonnegative definite, X_p, $p \times n$, where the columns are i.i.d. copies of the random vector $x_p \in \mathbb{R}^p$, isotropically distributed (that is, the distribution of x_p is the same as the distribution of O_px_p for any $O_p \in \mathcal{O}_p$). It is assumed that the moments of the e.d.f. of the eigenvalues of T_p satisfy the same properties as in [29]. The assumptions on the isotropically distributed x_p for increasing p is the following (as specified in [5]). With $L_p(\cdot)$ denoting the distribution function of $\|x_p\|$ ($\|\cdot\|$ denoting Euclidean norm), it is assumed (i) $\lim_{p\to\infty} \int_0^\infty r^2 dL(_p(r) < \infty$ for all p, (ii) there exists a function $R(\cdot)$ for which

$$\int_{R(p)}^\infty r^2 dL_p(r) = o(p), \quad \text{as } p \to \infty, \quad \lim_{p\to\infty} \left\{ \int_0^{R(p)} r^{2k} dL_p(r) \Big/ p^k \right\} = C_k \quad \text{exists for } k \in \{1, 2, \ldots\},$$

with

$$\sum_{k=1}^{\infty} (C_{2k}H_{2k})^{-1/2k} = \infty.$$

Then F^{A_p}, the e.d.f. of the eigenvalues of A_p, converges in probability weakly to a nonrandom distribution function, having moments explicitly given in terms of the H_k and the C_k.

The fourth and last time Professor Krishnaiah and JS met up was at the Sixth International Symposium on Multivariate Analysis in July 1983, in Pittsburgh. By this time we were on a first name basis (he was known as "Kris" to his friends). It was most memorable for JS mainly because he finally got to meet Prof. Jonsson, while he, YY and JS spent a good deal of time together. Another memorable event during this time was attending a dinner party in the home of Professor Krishnaiah.

In June 1984 YY and JS were together at a random matrix conference in Bowdoin Maine. He told JS that he, ZB, and Professor Krishnaiah were working on showing the largest eigenvalue of $B_p = (1/n)X_pX_p^T$ with the entries, X of X_p i.i.d. standardized, with finite fourth moment, and coming from one doubly infinite array of random variables, $p/n \to y > 0$, converges a.s. to $(1 + \sqrt{y})^2$, the right endpoint of the Marčenko–Pastur distribution. This extends the work of Suart Geman, who showed the a.s. convergence under the assumption of a certain growth condition on the moments of X [10]. This will be the last work Professor Krishnaiah would be involved in on large dimensional random matrices [28]. At the meeting YY also told JS that his student ZB would be visiting Pittsburgh soon.

ZB, arriving at the Center in fall 1984, remembers vividly his tenure at the University of Pittsburgh, especially his fellow colleagues at the Center. Professor Xiru Chen was one among 13 visitors Professor Krishnaiah had invited to the Center, along with ZB. Professor Chen is well-known in China as a strong mathematician and statistician, who was also co-supervisor of ZB's Ph.D. thesis. At one time ZB jokingly asked Professor Chen as to who he thought has the stronger abilities, he or Professor Krishnaiah. Upon asking ZB for more details, he mentioned that Professor Krishnaiah has the capability to give a different question to each of the visitors, who collectively had a broad range of knowledge and different areas of expertise, displaying his own extensive scope of understanding of the sciences. Professor Chen conceded that Professor Krishnaiah was stronger.

ZB recalled that Professor Krishnaiah was not only concerned with research activities of all his visitors, but also with their affairs of everyday life. Once ZB developed a backache and immediately informed Professor Krishnaiah that he did not have health insurance. He asked his secretary to buy insurance immediately for ZB and asked his Chinese student, Mr. Yijun Ding, to bring ZB to see the doctor (because ZB's English was not good enough to explain details to doctors, at that time). Next year (1985), when Dr. Lincheng Zhao arrived at the Center, Professor Krishnaiah asked his secretary to buy health insurance for him immediately. Because Dr. Zhao's oral English was not sufficient enough to talk to the secretary, Professor Krishnaiah asked ZB to talk to the secretary. After two weeks, upon learning Dr. Zhao had still not gotten health insurance, ZB asked the secretary why. She said: "it's not my business". Then ZB said to Professor Krishnaiah:

"the secretary said it's not my business" (interpreted as "it's none of my business"). ZB did not know why he became very unhappy, and he said to repeat it again. ZB said it once more. Professor Krishnaiah suddenly stood up and rushed to the secretary's office. And he said to the secretary: "you don't need to come tomorrow". And he said to his other secretaries: "All of my visitors are important people, you shouldn't be impolite to any one of them!" In the evening, ZB told the story to his roommate Zhaoben Fang. Dr. Fang said: "you made a big mistake in English! You should have said 'it's not *her* business'. Professor Krishnaiah must have understood the secretary to have said: 'it's none of *your* business'. This is a very impolite English sentence!" ZB reacted "Ah! my god!" He didn't really know the big difference is between the two sentences at that time. He apologized to the secretary the next day.

After one month of his arrival at the center, ZB asked Professor Krishnaiah to have a phone installed in his office, Professor Krishnaiah said it is not easy to get the phone cost reimbursed, but he can install a phone at his home, and can raise ZB's salary as compensation. Then, from the third month, ZB's salary increased from $500/month to $700/month.

In October 1984, YY's daughter, Yin Yin, talked about ZB to her advisor, Professor Raymond Carroll. He then called Professor Krishnaiah about ZB. He inferred that ZB might want to look for a new position in order to make more money than at the Center. Professor Krishnaiah called his secretary from the hospital (his throat problems were just discovered at that time) and asked her to file an application for the Carnegie-Mellon Post-doctorialship funding committee at the University of Pittsburgh. The application was denied in March 1985. By this time, Professor Krishnaiah was going back and forth to the hospital for periodical checkups on his throat. After several rounds of fighting with the funding committee, he talked to ZB: "I know I pay you less. But it is not because I don't have enough money nor I don't want to pay you more. The reason I cannot pay more is that my funds are restricted by military regulations. It cannot be paid to people from communist countries. I pay my money to the Indian students, and then collect their scholarships from the university. Then I can only use this limited funds to pay all of you from China. Please give me two weeks, I will file a proposal to the grant office of the Air Force, and if approved, I will increase your salary to $1500/month. If I cannot get the approval, then you can find a job elsewhere. I just can't pay you more at this time". After one week, he came to the office and happily said that he got the money. The Air force approved. ZB's salary was increased from $700/month to $1500/month, beginning in August 1985. Also due to this approval, all the Chinese scholars at the Center, Dr. Lincheng Zhao, Baiqi Miao, Prof. Xiru Chen, received higher initial payments than ZB.

Over the years Professor Krishnaiah and JS corresponded and spoke on the phone a great deal. He was a big supporter of JS's work on eigenvector behavior, with two papers of JS appearing in JMVA. He told JS the times National Science Foundation asked him to review proposals, submitting very strong evaluations. He once told JS he was editing a series of books, "Handbook on Statistics" and there will be a volume devoted to large dimensional random matrices. This did not happen. His last letter to JS is dated April 29, 1987. In it he asked JS to write a letter of recommendation for ZB. JS sent off the letter on May 19, and on July 24, received a note from C.R. Rao, acknowledging the letter, and adding "Kris is not well enough to handle this case. I am writing on his behalf. Thank you".

Since the beginning of 1987, Professor Krishnaiah developed throat cancer. The disease was getting worse and worse. He was feeding from a long tube through his nose. But he was still working on the journal. Every week, ZB was accompanying Mr. Young-Nien Sun (the computer manager of the Center, a Ph.D. student from the University of Pittsburgh's ECE department), who drove his car and brought a big box of letters from Professor Krishnaiah's secretaries to see him. He laid down on his bed and his wife read for him the letters from authors or his friends and the drafts of replies typed by his secretaries. If he thought it was okay, he answered by a nod and his wife would sign for him. Otherwise, he would say something and his wife would make changes on the draft and Mr. Sun and ZB would bring them back to the secretaries to make changes. His wife seemed to be tearing, but it never interrupted his work.

JS received a phone call from YY on August 1, 1987 of Professor Krishnaiah's passing.

For JS, YY, and ZB it was a tremendous loss. But because of Professor Krishnaiah's close association to the three of them, there arose a good deal of work between YY and ZB and between ZB and JS on random matrices. There is even one paper co-authored by the three of them [4]. His work and influence will live on.

3. Contributions to image reconstruction

The CT scanner was invented by Godfrey Newbold Hounsfield 1n 1972 and a Nobel prize was awarded to him in 1979. The principle of the machine is simple, just an inverse-Fourier transform. Consider a cross-section of a human's body, a two-dimensional distribution density, It can be easily reconstructed and visualized if its Fourier transform can be observed. Suppose the density is $p(x,y)$. Then the Fourier transform is $g(t,s)=\int e^{i(tx+sy)}p(x,y)dxdy$. Given a direction θ, write $t=\rho\cos\theta$ and $s=\rho\sin\theta$. Making a variable change $u=x\cos\theta+y\sin\theta$ and $v=-x\sin\theta+y\cos\theta$. Then

$$g(t,s)=h(\rho)=\int e^{i\rho u}\tilde{p}(u,v)dudv=\int e^{i\rho u}\bar{p}(u)du,\quad \bar{p}(u)=\int \tilde{p}(u,v)dv$$

which is the projection of p along θ and $h(\rho)$ is the Fourier transform of the projection $\bar{p}$. Because the projection is observable and hence the Fourier transform of $h(\rho)$ is estimable, consequently the two dimensional Fourier transform is estimable, if the projections can be made along all directions θ.

In medical science, the heart, especially the shape of the left ventricle, is an important subject for cardiologists. Although the CT scanner is a powerful instrument for observing pathological change in the body, it does not work for moving objects. Thus, the Siemens Gammasonics Inc. proposed the problem: how to extend the CT scanner to the left ventricle. Then, the cardiologist, Professor P. S. Reddy, along with Professor Krishnaiah and C.R. Rao, proposed to use two orthogonal X-ray pictures to reconstruct the left ventricle, and obtained a sizeable grant from Siemens Inc. of several hundreds of millions of dollars. Professor Reddy gave three hundred thousands dollars after overhead per quarter to the Center. Since Dr. Zhao arrived at the center, he and ZB devoted significant effort into the project. It is hard to say the project is statistical or mathematical. But it is interesting and is related to their academic survival; their stipends are all coming from this grant. ZB realized how Professor Krishnaiah had wide interests in scientific developments and broad general knowledge. This also taught ZB the importance of research toward real applications.

The commonly used methods assumed the cross section to be close to an ellipse [8], located between two end points of each projection profile as an axis of the ellipse. In 1983, Eiho et al. [9] refined the construction of ventricular cross section by using three projection profiles under the same assumption of an elliptic shape. These algorithms are simple, but the reconstructed cross section is far from a real ventricular cross section, which is usually not elliptical. Chang and Chow [6,7] proposed another algorithm to specially reconstruct the shape of the ventricle, by assuming the cross section of a ventricle to be a connected region which is convex and symmetric with respect to the geometrical center of the cross section. After applying their algorithm, the reconstructed cross sections can be selected from a few possible solutions. However, a ventricular cross section is in general neither convex nor symmetric [16,25].

In using the data on two orthogonal X-ray pictures taken in time, the group's strategy was in how to reconstruct the image of the object which has been cut into slices. They used the midpoints of the slices to connect them together and developed an algorithm to correctly align the midpoints from slice to slice by comparing the two orthogonal projections. Details of the reconstruction were published in [2] and the consistency of the procedure was theoretically proven in [3] under minor regular conditions. The algorithm has been tested on computer-synthesized projection data as well as real X-ray pictures with good results.

4. Contributions to GIC of model selection

It is well known that if a model is under-specified the data would not be well fitted by the model and if the model is over specified the efficiency becomes low because much information would be wasted by the estimation of extra parameters. In 1972, H. Akaike [1] proposed his famous model selection criterion, AIC. The AIC is minus two times the logarithm of the maximum likelihood (the information) plus two times the number of parameters (the penalty). The AIC received much attention, resulting in a fair amount of application in statistics. A deeper theoretical research on AIC found that there is a positive probability that the AIC over-specifies the true model. In 1978, G. Schwarz [17] proposed his well known BIC from the view point to minimize the mis-specifying probability for a Bayesian model. The BIC changes the penalty in AIC as $\log n$ times the number of parameters. It was proven that the BIC was strongly consistent, but sometimes it under-specifies the true model. In 1979, H.J. Hannan and B.G. Quinn [12] changed the penalty as $\log\log n$ times the number of parameters for an autoregressive model and showed that the information selection criterion is strongly consistent and that $\log\log n$ cannot be further made smaller.

Since then, the question as to how to set up the information criteria became a hot topic in the literature. In late 1985, after Dr. Zhao came and joined the Center, Professor Krishinaiah proposed this problem to him and ZB. They proposed the GIC (general information criterion), that changes the penalty as C_n times the number of parameters, satisfying $C_n/n \to 0$ and $C_n/\log\log n \to \infty$. They showed that the GIC is strongly consistent. The publications [32,33] became highly cited in the literature. Based on the idea of GIC, many data-oriented penalty rules were proposed in the literature.

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References

[1] H. Akaike, Information theory and an extension of the maximum likelihood principle, in: Proceedings of the Second International Symposium on Information Theory, Suppl. to Problems of Control and Information Theory, 1972, pp. 267–281.
[2] Z.D. Bai, P.R. Krishnaiah, C.R. Rao, P.S. Reddy, Y.N. Sun, L.C. Zhao, Reconstruction of the left ventricle from two orthogonal projections, Comput. Vis. Graph. Image Process. 47 (1989) 165–188.
[3] Z.D. Bai, P.R. Krishnaiah, C.R. Rao, P.S. Reddy, Y.N. Sun, L.C. Zhao, Reconstruction of the shape and size of objects from two orthogonal projections, Math. Comput. Model. 12 (1989) 267–275.
[4] Z.D. Bai, J.W. Silverstein, Y.Q. Yin, A note on the largest eigenvalue of a large-dimensional sample covariance matrix, J. Multivariate Anal. 26 (1988) 166–168.
[5] Z.D. Bai, Y.Q. Yin, P.R. Krishnaiah, On limiting spectral distribution of product of two random matrices when the underlying distribution is isotropic, J. Multivariate Anal. 19 (1986) 189–200.
[6] S.K. Chang, The reconstruction of binary patterns from their projections, Commun. ACM 14 (1971) 21–25.
[7] S.K. Chang, C.K. Chow, The reconstruction of binary patterns from two orthogonal projections and its application to cardiac cineangiography, IEEE Trans. Comput. C 22 (1973) 18–28.
[8] H.T. Dodge, H. Sandler, D.W. Ballew, J.D. Lord Jr., The use of biplane angiocardiography for the measurement of left ventricular volume in man, Amer. Heart J. 60 (1960) 762–776.

[9] S. Eiho, M. Kuwahara, K. Shimura, M. Wada, M. Ohta, T. Kozuka, Reconstruction of the left ventricle from X-ray cineangiocardiograms with a rotating arm, in: Computers in Cardiology 1983, IEEE Computer Society, 1984, pp. 63–67.
[10] S. Geman, A limit theorem for the norm of random matrices, Ann. Probab. 8 (1980) 252–261.
[11] U. Grenander, J.W. Silverstein, Spectral analysis of networks with random topologies, SIAM J. Appl. Math. 32 (1977) 499–519.
[12] E.J. Hannan, B.G. Quinn, The determination of the order of an autoregression, J. R. Stat. Soc. Ser. B Stat. Methodol. 41 (1979) 190–195.
[13] D. Jonsson, Some limit theorems for the eigenvalues of a sample covariance matrix, J. Multivariate Anal. 12 (1982) 1–38.
[14] P.R. Krishnaiah, T.C. Chang, On the exact distributions of the extreme roots of the Wishart and MANOVA matrices, J. Multivariate Anal. 1 (1971) 108–117.
[15] V.A. Marčenko, L.A. Pastur, Distribution of eigenvalues for some sets of random matrices, Math. USSR-Sb 1 (1967) 457–483.
[16] D.G.W. Onnasch, P.H. Heintzen, A new approach for the reconstruction of the right and left ventricular form from biplane angiocardiographic recordings, in: Computers in Cardiology 1976, IEEE Computer Society, 1976, pp. 67–73.
[17] G. Schwarz, Estimating the dimension of a model, Ann. Statist. 6 (1978) 461–464.
[18] J.W. Silverstein, On the randomness of eigenvectors generated from networks with random topologies, SIAM J. Appl. Math. 37 (1979) 235–245.
[19] J.W. Silverstein, Describing the behavior of eigenvectors of random matrices using sequences of measures on orthogonal groups, SIAM J. Math. Anal. 12 (1981) 274–281.
[20] J.W. Silverstein, Comments on a result of Yin, Bai, and Krishnaiah for large dimensional multivariate F matrices, J. Multivariate Anal. 15 (1984) 166–168.
[21] J.W. Silverstein, Some limit theorem on the eigenvectors of large dimensional sample covariance matrices, J. Multivariate Anal. 15 (1984) 295–324.
[22] J.W. Silverstein, The limiting eigenvalue distribution of a multivariate F matrix, SIAM J. Math. Anal. 16 (1985) 641–646.
[23] J.W. Silverstein, On the eigenvectors of large dimensional sample covariance matrices, J. Multivariate Anal. 30 (1989) 1–16.
[24] J.W. Silverstein, Weak convergence of random functions defined by the eigenvectors of sample covariance matrices, Ann. Probab. 18 (1990) 1174–1194.
[25] C.H. Slump, J.J. Gerbrands, A network flow approach to reconstruction of the left ventricle from two projections, Comput. Graph. Image Process. 18 (1982) 18–36.
[26] K.W. Wachter, The strong limits of random matrix spectra for sample matrices of independent elements, Ann. Probab. 6 (1978) 1–18.
[27] Y.Q. Yin, Z.D. Bai, P.R. Krishnaiah, Limiting behavior of the eigenvalues of a multivariate F matrix, J. Multivariate Anal. 13 (1983) 508–516.
[28] Y.Q. Yin, Z.D. Bai, P.R. Krishnaiah, On the limit of the largest eigenvalue of the large dimensional sample covariance matrix, Probab. Theory Related Fields 78 (1988) 509–521.
[29] Y.Q. Yin, P.R. Krishnaiah, Limit theorems for the eigenvalues of product of two random matrices, J. Multivariate Anal. 13 (1983) 489–507.
[30] Y.Q. Yin, P.R. Krishnaiah, Limit theorem for the eigenvalues of the sample covariance matrix when the underlying distribution is isotropic, Teoriya Veroyatnostei I Ee Primeneniya (Theory of Probability and Its Applications: English Translation Published By SIAM) 30 (1985) 810–816.
[31] Y.Q. Yin, P.R. Krishnaiah, Limit theorems for the eigenvalues of product of large dimensional random matrices when the underlying distribution is isotropic Teoriya Veroyatnostei i ee Primeneniya, Theory Probab. Appl. 31 (1986) 394–398, English translation published by SIAM.
[32] L.C. Zhao, P.R. Krishnaiah, Z.D. Bai, On detection of number of signals in presence of white noise, J. Multivariate Anal. 20 (1986) 1–25.
[33] L.C. Zhao, P.R. Krishnaiah, Z.D. Bai, On detection of number of signals when the noise covariance matrix is arbitrary, J. Multivariate Anal. 20 (1986) 26–49.